Indian J Phys. **82** (11), 1563-1575 (2008)

Interaction of an electric charge in the radial field of a heavy dyon

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Received 29 February 2008, accepted 8 July 2008

Abstract Considering one scalar potential and one vector potential for the interaction of an electric charge in the radial field of a dyon we have undertaken the study of equation of motion through Lagrangian and Hamiltonian formulations The relation of magnetic field with the vector potential has been given a new interpretation which leads to an additional angular momentum in a direction transverse to radial direction which could be understood as spinning top like angular momentum In presence of this additional angular momentum, the commutation relations between the components of total angular momentum and other related operators of the system take spherically symmetric forms The eigen value problem of total angular momentum and Hamiltonian operator has been also analysed

Keywords . Dyon, vector potential, topspin

PACS No. 14 80 Hv

1. Introduction

The subject of magnetic charge has been of great interest since the ingenious work of Dirac [1] in order to make the Maxwell's equations symmetric and to explain the observed quantization of electric charge Later Schwinger [2, 3] and Zwanziger [4] extended this idea to dually charged particles namely dyons and successfully developed the quantum field theory of these particles. Today magnetic monopoles and dyons have become the intrinsic parts of all current grand unified theories [5] with enormous potential importance in connection with their roles in catalyzing proton decay [6, 7], the quark confinement problem of QCD [8,9] and CP violation [10].

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In this paper we have studied the motion of an electric charge in the radial field of a fixed dyon Assigning new concept to vector potential we construct the Lagrangian and Hamiltonian of the system which yield the correct equations of motion We have also undertaken the study of angular momentum operator and canonical quantization of the system in consideration The string singularity in vector potential is completely absent in this formulation, moreover being a non relativistic treatment the intrinsic spin of the particles has not been taken into consideration The fact, that for the electric charge in any kind of magnetic field and *vice versa* the force must be Lorentz force, has been made the central point in this treatment

2. Behaviour of vector potential in radial magnetic field

The usual electrodynamics, in absence of magnetic charge and corresponding current density for all situations of electromagnetic fields uses the relations,

$$\boldsymbol{A} = \frac{1}{2}\boldsymbol{H}\times\boldsymbol{r} \tag{2 1}$$

and

$$\nabla\times\boldsymbol{A} = \boldsymbol{H} \ , \tag{2 2}$$

while in the case of radial magnetic field the relation (2 1) looses meaning and so happens with the relation (2 2) Therefore for a non vanishing vector potential we use the following option to define the vector potential as,

$$\boldsymbol{A} = \frac{1}{2}\boldsymbol{H}\times\boldsymbol{r}_T \tag{2 3}$$

where $\boldsymbol{r}_T$ is a vector transverse to the vector $\boldsymbol{r}$ i e

$$r_T = \hat{i}\left(r_j - r_k\right) + \hat{j}\left(r_k - r_i\right) + \hat{k}\left(r_i - r_j\right), \tag{2 4}$$

so that,

$$\boldsymbol{r}\cdot\boldsymbol{r}_T = 0 , \tag{2 5}$$

and the field can be obtained as,

$$\nabla_T\times\boldsymbol{A} = \boldsymbol{H} \tag{2 6}$$

The realtion (2 6) could be easily obtained using, eq (2 3) for $\boldsymbol{A}$ and identity for vector triple product In the relation (2 6) $\boldsymbol{A}$ is a function of r_i, r_j, r_k while ∇_T is a transformed operator in the new relative coordinates ξ_i , ξ_j , ξ_k defined as,

$$\xi_i = r_j - r_k, \qquad \xi_j = r_k - r_i , \qquad \xi_k = r_i - r_j \tag{2 7}$$

In this new coordinate system using the derivative transformation,

$$\frac{\partial}{\partial r_i} = \frac{\partial}{\partial \xi_i}\frac{\partial \xi_i}{\partial r_i} + \frac{\partial}{\partial \xi_j}\frac{\partial \xi_j}{\partial r_i} + \frac{\partial}{\partial \xi_k}\frac{\partial \xi_k}{\partial r_i} \quad etc \tag{2.8}$$

we have the following transformations for derivatives

$$\left.\begin{aligned} \frac{\partial}{\partial r_i} &\Rightarrow \frac{\partial}{\partial \xi_k} - \frac{\partial}{\partial \xi_j}\,, \\ \frac{\partial}{\partial r_j} &\Rightarrow \frac{\partial}{\partial \xi_i} - \frac{\partial}{\partial \xi_k}\,, \\ \frac{\partial}{\partial r_k} &\Rightarrow \frac{\partial}{\partial \xi_j} - \frac{\partial}{\partial \xi_i}\,, \end{aligned}\right\} \tag{2.9}$$

so that the components of $\nabla_T \times \boldsymbol{A}$, where $\boldsymbol{A}$ is retained as a vector function of old coordinates r_i, r_j, r_k are obtained as,

$$\left.\begin{aligned} (\nabla_T \times \boldsymbol{A})_i &= \left(\frac{\partial}{\partial \xi_i} - \frac{\partial}{\partial \xi_k}\right)A_k - \left(\frac{\partial}{\partial \xi_j} - \frac{\partial}{\partial \xi_i}\right)A_j \\ (\nabla_T \times \boldsymbol{A})_j &= \left(\frac{\partial}{\partial \xi_j} - \frac{\partial}{\partial \xi_i}\right)A_i - \left(\frac{\partial}{\partial \xi_k} - \frac{\partial}{\partial \xi_j}\right)A_k\,, \\ (\nabla_T \times \boldsymbol{A})_k &= \left(\frac{\partial}{\partial \xi_k} - \frac{\partial}{\partial \xi_j}\right)A_j - \left(\frac{\partial}{\partial \xi_i} - \frac{\partial}{\partial \xi_k}\right)A_i \end{aligned}\right| \tag{2.10}$$

3. Lagrangian and force law

We consider a system of an electric charge e_1 in the radial field of a dyon having electric charge e_2 and magnetic charge g_2. To describe the interaction between the two we consider a scalar potential $\phi(\boldsymbol{r})$ and a vector potential $\boldsymbol{A}(\boldsymbol{r})$ describing the electric-electric (e_1, e_2) charge coupling and electric-magnetic (e_1, g_2) charge coupling respectively. In terms of these potentials the Lagrangian for the relative motion of electric charge in the field of heavy dyon could be written as,

$$L = \frac{1}{2}\sum m r_i^2 - e_1\phi(\boldsymbol{r}) + \frac{e_1}{c}\boldsymbol{v}\cdot\boldsymbol{A}(\boldsymbol{r})\,, \tag{3.1}$$

where m is the reduced mass of electric charge and $\phi(\boldsymbol{r})$ and $\boldsymbol{A}(\boldsymbol{r})$ are the functions of spatial coordinates $\boldsymbol{r}(r_i, r_j, r_k)$ only and not having explicit dependence on time This is the usual way of writing the lagrangian in present electrodynamics where all situations of electromagnetic fields are attributable to electric charge source and corresponding current density where the magnetic charges and corresponding currents are absent, but in our case in presence of magnetic charge and an electric charge interacting with it, the behaviour of vector potential will be different as demonstrated in section-2 The Lagrangian given by eq (3 1) yields,

$$p_i = \frac{\partial L}{\partial r_i} = mr_i + \frac{e_1}{c} A_i \tag{3 2}$$

and,

$$\frac{\partial L}{\partial r_i} = -e_1\phi_{,i} + \frac{e_1}{c}\left(r_j A_{j,i} + r_k A_{k,i}\right) \ , \tag{3 3}$$

where the symbols $\phi_{,i}$ and $A_{j,i}$ will imply the derivatives $(\partial\phi)/(\partial r_i)$ and $(\partial A_j)/(\partial r_i)$ *etc* Using the derivative transformation eqn (2 9) for $A_{j,i}$ and $A_{k,i}$ in (3 3) and substituting we get,

$$\frac{\partial L}{\partial r_i} = -e_1\phi_{,i} + \frac{e_1}{c}\left[r_j\left(\frac{\partial A_j}{\partial \xi_k} - \frac{\partial A_j}{\partial \xi_j}\right) + r_k\left(\frac{\partial A_k}{\partial \xi_k} - \frac{\partial A_k}{\partial \xi_j}\right)\right] \tag{3 4}$$

The total time derivative of eq (3 2) is,

$$\frac{d}{dt}\frac{\partial L}{\partial r_i} = mr_i + \frac{e_1}{c}\left[r_j A_{i,j} + r_k A_{i,k}\right] \tag{3 5}$$

Again transforming the derivatives $A_{i,j}$ and $A_{i,k}$ in eq (3 5) to ξ coordinates using eq (2 9) we get,

$$\frac{d}{dt}\frac{\partial L}{\partial r_i} = mr_i + \frac{e_1}{c}\left[r_j\left(\frac{\partial A_i}{\partial \xi_i} - \frac{\partial A_i}{\partial \xi_k}\right) + r_k\left(\frac{\partial A_i}{\partial \xi_j} - \frac{\partial A_i}{\partial \xi_i}\right)\right] \tag{3 6}$$

Substituting eq (3 6) and (3 4) in Euler Lagrange equation and combining the terms we get the equation of motion as,

$$mr_i = e_1\phi_{,i} + \frac{e_1}{c}\left[r_j\left(\nabla_T \times \boldsymbol{A}\right)_k - r_k\left(\nabla_T \times \boldsymbol{A}\right)_j\right]. \tag{3 7}$$

Similarly finding $m\ddot{r}_j$ and $m\ddot{r}_k$ and adding vectorily we get the equation of motion as

$$m\ddot{\boldsymbol{r}} = -e_1\nabla\phi + \frac{e_1}{c}\boldsymbol{v}\times(\nabla_T \times \boldsymbol{A}) \tag{3.8}$$

In our case

$$\left.\begin{aligned} \nabla_T \times \boldsymbol{A} = \boldsymbol{H} &= \frac{g_2}{r^3}\boldsymbol{r} \\ \text{and} \qquad -\nabla\phi &= \frac{e_2}{r^3}\boldsymbol{r} \end{aligned}\right\} \tag{3.9}$$

Finally the eq. of motion is obtained as

$$m\ddot{\boldsymbol{r}} = \alpha\frac{\boldsymbol{r}}{r^3} + \mu\frac{\boldsymbol{v}\times\boldsymbol{r}}{r^3}, \tag{3.10}$$

where $\alpha = e_1e_2$ and $\mu = \dfrac{e_1g_2}{c}$ (3.11)

are electric and magnetic coupling parameters respectively. The magnetic coupling parameter is constrained by the Dirac [1] quantization condition $(e_1g_2)/c \;\; \hbar$ (integer or half odd integer).

4. Hamiltonian and force law

Using the general definition

$$\mathcal{H} = \sum p_i\dot{r}_i - L \tag{4.1}$$

and replacing the canonical momenta $\boldsymbol{p}$ by $\pi = (\boldsymbol{p} - (e_1/c)\boldsymbol{A}(\boldsymbol{r}))$ we get the gauge invariant Hamiltonian of the system in consideration as

$$\mathcal{H} = \sum_{i=1}^{3}\frac{1}{2m}\left(p_i - \frac{e_1}{c}A_i\right)^2 + e_1\phi \tag{4.2}$$

where the components of kinetic momenta π are

$$\pi_i = \frac{1}{2m}\left(p_i - \frac{e_1}{c}A_i\right) \qquad etc. \tag{4.3}$$

Now the Hamilton's equatiosn yield,

$$\left.\begin{aligned} \dot{r}_i = \frac{\partial\mathcal{H}}{\partial p_i} &= \frac{1}{m}\left(p_i - \frac{e_1}{c}A_i\right) \\ \text{and similar equations for } \dot{r}_j &\text{ and } \dot{r}_k \end{aligned}\right\} \tag{4.4}$$

$$\left.\begin{array}{l} \text{as well as,} \quad p_i = -\dfrac{\partial \mathcal{H}}{\partial r_i} = \dfrac{e_1}{c}\left(r_j A_{j,i} + r_k A_{k,i}\right) - e_1 \phi_{,i} \\ \\ \text{and similar equations for } p_j \text{ and } p_k \end{array}\right\} \tag{4.5}$$

Taking the total time derivative of eq (4.4) and substituting the value of p_i in that equation and rearranging the terms we get,

$$m\ddot{r}_i = -e_1\phi_{,i} + \frac{e_1}{c}\left[\dot{r}_j\left(A_{j,i} - A_{i,j}\right) - \dot{r}_k\left(A_{i,k} - A_{k,i}\right)\right] \tag{4.6}$$

Transforming the derivatives in eqn (4.6) to (ξ_i, ξ_j, ξ_k) system with the help of eq (2.9) and interpreting the brackets in terms of the components of $\nabla_T \times \boldsymbol{A}$ with the help of eqs (2.10), eq (4.6) takes the form,

$$m\ddot{r}_i = -e_1\phi_{,i} + \frac{e_1}{c}\left[\dot{r}_j\left(\nabla_T \times A\right)_k - \dot{r}_k\left(\nabla_T \times A\right)_j\right] \tag{4.7}$$

Interpreting the components of $\nabla_T \times \boldsymbol{A}$ as the component of radial magnetic field and obtaining similar eqs for r_j and r_k coordinates and adding the equations vectorily, we get the final equation of motion as,

$$m\ddot{\boldsymbol{r}} = \alpha\frac{\boldsymbol{r}}{r^3} + \mu\frac{\boldsymbol{v} \times \boldsymbol{r}}{r^3} \tag{4.8}$$

where α and μ are given by eqn (3.11) The purpose of demonstrating eq (3.10) and (4.8) is to show the consistency of the relations (2.3) and (2.6) with the Lorentz force equations These equations of motion have been used by Schwinger *et al* [11] in case of classical non relativistic description of charge-monopole and dyon-dyon scattering using the concept of trajectory

5. Angular momentum operator

In consistence with the Lagrangian and Hamiltonian formulations, the gauge invariant angular momentum of the system of electric charge in the field of dyon could be written as,

$$\boldsymbol{J} = \boldsymbol{r} \times \boldsymbol{\pi} \tag{5.1}$$

where $\boldsymbol{\pi}$ is the kinetic momentum given by,

$$\boldsymbol{\pi} = \boldsymbol{p} - \frac{e_1}{c}\boldsymbol{A} \tag{5.2}$$

Therefore,

$$\boldsymbol{J} = \boldsymbol{r} \times \boldsymbol{p} - \frac{e_1}{c} \boldsymbol{r} \times \boldsymbol{A} . \tag{5 3}$$

Using the value of $\boldsymbol{A}$ as $\boldsymbol{A} = 1/2\, \boldsymbol{H} \times \boldsymbol{r}_T$ from eq (2 3) and simplifying the vector triple product of second factor in eq. (5 3) as,

$$\frac{1}{2} \boldsymbol{r} \times \boldsymbol{H} \times \boldsymbol{r}_T = \frac{1}{2}\left[(\boldsymbol{r} \cdot \boldsymbol{r}_T)\boldsymbol{H} - (\boldsymbol{r} \cdot \boldsymbol{H})\boldsymbol{r}_T\right],$$

with the conditions,

$$\boldsymbol{r} \cdot \boldsymbol{r}_T = 0 \quad \text{and} \quad \boldsymbol{H} = \frac{g_2}{r^3} \boldsymbol{r} ,$$

the total angular momentum is obtained as,

$$\boldsymbol{J} = \boldsymbol{L} + \frac{\mu}{2} \frac{\boldsymbol{r}_T}{r} , \tag{5 4}$$

where μ is the magnetic coupling parameter defined in eq (3 11) The plus sign in the mid of eq (5 4) will change to negative for the bound state of the system

It may be concluded from eq (5 4) that the total angular momentum possesses an extra angular momentum $(\mu/2)\, \hat{r}_T$ besides the orbital and intrinsic spin angular momentum which arises due to coupling of the field of magnetic charge in dyon and the field of electric charge in motion The direction of the extra angular momentum is transverse to the radial direction therefore could be conceived like the angular momentum of spinning top instead of helicity like as reported by Schwinger *et al* [11] and Zwanziger [12]

Reading the components of $\boldsymbol{r}_T$ from eq (2 4) and using the basic commutators,

$$\left.\begin{aligned} \left[L_i, r_j\right] &= i\hbar \varepsilon_{ijk} r_k , \\ \left[L_i, L_j\right] &= i\hbar \varepsilon_{ijk} L_k , \\ \left[L_i, (\boldsymbol{r}_T)_j\right] &= -i\hbar r_j , \\ \left[L_j, (\boldsymbol{r}_T)_i\right] &= -i\hbar r_i , \end{aligned}\right\} \tag{5 5}$$

we can easily obtain the following spherically symmetric commutation relation between the components of total angular momentum,

$$\left[J_i, J_j\right] = i\hbar \varepsilon_{ijk} J_k , \tag{5 6}$$

which follows,

$$\left[J^2, J_i\right] = \left[J^2, J_j\right] = \left[J^2, J_k\right] = 0 \tag{5 7}$$

If we use the basic commutators between the particle coordinates and the components of canonical momenta as well as with the potential components of the field as,

$$\left.\begin{aligned}
&\left[r_i, r_j\right] = \left[p_i, p_j\right] = 0 \quad , && \text{(i)}\\
&\left[r_i, p_j\right] = i\hbar\, \delta_{ij} \quad , && \text{(ii)}\\
&\left[A_i, p_j\right] = 0 \quad , && \text{(iii)}\\
&\left[\phi_i, p_j\right] = 0 \quad , && \text{(iv)}
\end{aligned}\right| \tag{5 8}$$

we get the following gauge invariant and rotationally symmetric commutation relations

$$\begin{aligned}
&\left[r_i, \pi_j\right] = i\hbar\, \delta_{ij} \quad , && \text{(i)}\\
&\left[\pi_i, \pi_j\right] = 0 && \text{(ii)}\\
&\left[J_i, \pi_j\right] = i\hbar\varepsilon_{ijk}\pi_k \quad , && \text{(iii)}\\
&\left[J_i, r_j\right] = i\hbar\varepsilon_{ijk} r_k \quad , && \text{(iv)}
\end{aligned} \tag{5 9}$$

and $$\left[J_i, J_j\right] = \left[(\boldsymbol{r}\times\boldsymbol{\pi})_i, (\boldsymbol{r}\times\boldsymbol{\pi})_j\right] = i\hbar\varepsilon_{ijk} J_k \qquad \text{(v)}$$

In deriving the commutation relation (5 6) and same as (5 9 v) we have adopted two different approaches The relation (5 6) depend on the particular representation eq (2 3) of the vector potential $\boldsymbol{A}$ in terms of the transverse vector $\boldsymbol{r}_T$ while (5 9 v) depend on the basic commutation (5 8 iii) followed by the components of canonical momentum and that of vector potential The two approaches seem to be complementary to each other and hence justifies the basic commutator (5 8 iii) in one side and the presence of additional angular momentum as topspin angular momentum (5 4), on the other

Expressing the angular momentum (5 4) as the sum of two angular momenta *i e*

$$\boldsymbol{J} = \boldsymbol{J}_1 + \boldsymbol{J}_2 \,, \tag{5 10}$$

we find the following for the projections of J_1 and J_2 along $\hat{r}$ and $\hat{r}_T$ directions,

$$\hat{r}\cdot J_1 = \hat{r}\cdot J_2 = 0 \quad , \qquad \text{(i)}$$

$$\hat{r}_T\cdot J_1 \neq 0 \quad , \qquad \text{(ii)} \qquad (5.11)$$

and

$$\hat{r}_T\cdot J_1 = \mu \qquad \text{(iii)}$$

Now expressing the space quantization of two angular momenta along the reference z-axis as $(J_1)_z = m_l\hbar$ and $(J_2)_z = m_\mu\hbar$ respectively where m_l ranges from l to $-l$ in the step of one and m_μ ranges from $\mu/2$ to $-(\mu/2)$ in the step of one, the resulting $m_l + m_\mu$ values would predict the total angular momentum quantum number j as

$$j = \left| l + \frac{\mu}{2} \right| \ldots \left| l - \frac{\mu}{2} \right| \qquad (5.12)$$

6 Bound state and energy eigen values

We now introduce the conditions of $\alpha = -e_1e_2$ and $\mu = -e_1g_2$ in our treatment for the bound states to exist in case of electric charge-dyon system. We extend the Pauli method [13] applicable to the systems of one scalar potential to the system of one scalar potential and one vector potential.

Writing the Runze-lenz vector as,

$$\boldsymbol{R} = \frac{1}{2m}(\boldsymbol{J}\times\boldsymbol{\pi} - \boldsymbol{\pi}\times\boldsymbol{J}) + \alpha\frac{\boldsymbol{r}}{r}, \qquad (6.1)$$

which could be written in the following simplified form,

$$\boldsymbol{R} = \frac{1}{m}(\boldsymbol{J}\times\boldsymbol{\pi} - i\hbar\boldsymbol{\pi}) + \alpha\frac{\boldsymbol{r}}{r} \qquad (6.2)$$

Using the commutators (5.9.i) and (5.9.ii) as well as the following relations

$$\left.\begin{aligned}
&\left[(\boldsymbol{J}\times\boldsymbol{\pi})_i, \pi_j\right] + \left[\pi_i, (\boldsymbol{J}\times\boldsymbol{\pi})_j\right] = 0, && \text{(i)}\\
&\left[(\boldsymbol{J}\times\boldsymbol{\pi})_i, (\boldsymbol{J}\times\boldsymbol{\pi})_j\right] = -i\hbar\pi^2 J_k, && \text{(ii)}\\
&\left[(\boldsymbol{J}\times\boldsymbol{\pi})_i, \frac{r_j}{r}\right] + \left[\frac{r_i}{r}, (\boldsymbol{J}\times\boldsymbol{\pi})_j\right] = 2i\hbar\frac{J_k}{r} && \text{(iii)}
\end{aligned}\right\} \qquad (6.3)$$

we obtain the commutation relation,

$$\left[R_i\ R_j\right] = i\hbar\varepsilon_{ijk}J_k \left(-\frac{2\mathcal{H}}{m} \right) \tag{6 4}$$

Where $\mathcal{H}$ is the Hamiltonian operator described by equation

$$\mathcal{H} = \sum_{i=1}^{3} \frac{\pi_i^2}{2m} + \frac{\alpha}{r}, \tag{6 5}$$

with

The commutation relations (5 9) also yield,

$$\left[J_i, R_j\right] = i\hbar\varepsilon_{ijk}R_k\ , \tag{6 6}$$

$$\left[\boldsymbol{J}, \mathcal{H}\right] - \left[\boldsymbol{R}, \mathcal{H}\right] = 0 \tag{6 7}$$

Further considering the identities,

$$\left.\begin{aligned}
&(\boldsymbol{J}\times\boldsymbol{\pi})\ (\boldsymbol{J}\times\boldsymbol{\pi}) = J^2\pi^2 &&,\\
&\frac{\boldsymbol{r}}{r}\ (\boldsymbol{J}\times\boldsymbol{\pi}) + (\boldsymbol{J}\times\boldsymbol{\pi})\ \frac{\boldsymbol{r}}{r} = -\frac{2J^2}{r} + 2i\hbar\frac{\boldsymbol{r}}{r}\ \boldsymbol{\pi} &&,\\
&\boldsymbol{\pi}\ \frac{\boldsymbol{r}}{r} = \frac{\boldsymbol{r}}{r}\ \boldsymbol{\pi} - 2i\frac{\hbar}{r} &&,\\
&\boldsymbol{\pi}\ \boldsymbol{J} = \boldsymbol{J}\ \boldsymbol{\pi} = 0 &&,\\
&\boldsymbol{\pi}\ (\boldsymbol{J}\times\boldsymbol{\pi}) + (\boldsymbol{J}\times\boldsymbol{\pi})\ \boldsymbol{\pi} = 2i\hbar\pi^2 &&,
\end{aligned}\right\} \tag{6 8}$$

we get,

$$\boldsymbol{R}\quad \boldsymbol{J} = \boldsymbol{J}\quad \boldsymbol{R} = 0 \tag{6 9}$$

and

$$R^2 = \boldsymbol{R}\ \boldsymbol{R} = \alpha^2 + \frac{2\mathcal{H}}{m}\left(J^2 + \hbar^2\right) \tag{6 10}$$

Now we define the new operators,

$$\tau_1 = \frac{\boldsymbol{J}}{2\hbar} + \frac{1}{2\hbar}\left(-\frac{m}{2\mathcal{H}}\right)^{1/2}\boldsymbol{R}$$

and $$\tau_2 = \frac{\boldsymbol{J}}{2\hbar} - \frac{1}{2\hbar}\left(-\frac{m}{2\mathcal{H}}\right)^{1/2} \boldsymbol{R} \tag{6.11}$$

The operators τ_1 and τ_2 could be identified as new independent angular momentum operators following the Lie algebra

$$\begin{aligned} \boldsymbol{\tau}_1 \times \boldsymbol{\tau}_1 &= i\hbar\boldsymbol{\tau}_1 \\ \boldsymbol{\tau}_2 \times \boldsymbol{\tau}_2 &= i\hbar\boldsymbol{\tau}_2 \\ \boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2 &= 0 \end{aligned} \tag{6.12}$$

and because of the eqs (6.9) and (6.10) we obtain,

$$\tau_1^2 = \tau_2^2 = -\frac{1}{4}\left[\frac{1}{\mathcal{H}}\frac{m\alpha^2}{2\hbar^2} + 1\right] \tag{6.13}$$

The Lie algebra (6.12) and the relation (6.13) suggest a set of simultaneous eigen states for $\tau_1^2, \tau_2^2, \tau_{1k}, \tau_{2k}$ as,

$$|\phi\rangle = |\tau_1, \tau_2, \tau_{1k}, \tau_{2k}\rangle, \tag{6.14}$$

which is also the eigen state of Hamiltonian operator $\mathcal{H}$ because of,

$$[\tau_1, \mathcal{H}] = [\tau_2, \mathcal{H}] = 0 \tag{6.15}$$

The resulting eigen value equations are,

$$\left.\begin{aligned} \tau_1^2|\phi\rangle = \tau_2^2|\phi\rangle &= \frac{k_1}{2}\left(\frac{k_1}{2}+1\right)|\phi\rangle, && \text{(i)} \\ \tau_{1k}|\phi\rangle &= m_1|\phi\rangle, && \text{(ii)} \\ \tau_{2k}|\phi\rangle &= m_2|\phi\rangle, && \text{(iii)} \end{aligned}\right\} \tag{6.16}$$

where

$$\left.\begin{aligned} -\frac{k_1}{2} \le m_1 \le \frac{k_1}{2} \\ \text{and} \quad -\frac{k_2}{2} \le m_2 \le \frac{k_2}{2} \end{aligned}\right\} \tag{6.17}$$

Because of the construction of τ_1 and τ_2 the eigen values will be simply numbers not in $\hbar$ Using the relations (6 13) and eigen value eq (6 16) i) we get the eigen value equation of Hamiltonian as,

$$\mathcal{H}|\phi\rangle = E_n|\phi\rangle, \tag{6 18}$$

where

$$E_n = -m\frac{\alpha^2}{2\hbar^2}\ \frac{1}{r^2}, \tag{6 19}$$

in which we have set $k + 1 = n$, and α is the electric coupling parameter The quantum numbers are n m_1, m_2 and energy eigen values are hydrogen atom like as expected because coupling between electric charge in motion and magnetic charge of dyon does not add and extra energy in the system because of the nature of Lorentz force

7. Conclusions

Usual electrodynamics assumes the absence of magnetic charge, where the presence of any sort of magnetic field is expressed as Curl of a vector potential (eq 2 2) which is reciprocally related with the field as eq (2 1) Considering the problem of an electric charge in the radial field of dyon we have come to the conclusion that the current of electric charge couples with the radial field of magnetic charge of dyon through a vector potential given by eq (2 3) which is related to the field as eq (2 6) This notion is consistent with the Lagrangian and Hamiltonian formulations of equation of motion as eqn (3 10) and eq (4 8), as well as gives rise to an additional angular momentum of electric charge transverse to radial direction besides the orbital and intrinsic spin angular momentum and hence could be conceived like top spin angular momentum In presence of this spinning top like angular momentum the total angular momentum becomes spherically symmetric and leads to the possibility of formation of bound state of the system The analysis of eigen value problem of Hamiltonian yields hydrogen atom like energies

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